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An Integer Programming Model for Solving Periodic Heterogeneous Vehicle Routing Problem with Time Windows in Food Material Distribution

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ABSTRACT*:* Vehicle routing problem (VRP) as a well known model in distribution systems, is to seek which route should be travelled by the fleet of homogeny vehicle such that to minimize costs. However, whenever the distribution company needs to deliver a large number of pack such as food material, it is more practical to use vehicles with different types and capacity. This kind of variant is called heterogeneous VRP . It is assumed that customers must be served with a certain frequency and according to a given schedule and they should receive a fixed quantity at each visit.. This paper considers a food material logistic distribution problem in which it is assumed that customers must be served with a certain frequency and according to a given schedule and they should receive a fixed quantity as ordered at each visit. Now the problem has a suitable structure as the periodic HVRP. The problem is then modeled as a mixed integer program. A neighborhood search hybrid approach is developed for solving the result model.

KEYWORDS*:* Modeling, Distribution, Neighborhood search, Mixed Integer Programming, Hybrid approach

I. INTRODUCTION

The vehicle routing problem (VRP) is an interesting topic in operational decision for logistic distribution planning. In the realm of operations research, this topic had created a real challenge to be solved due to its combinatorial nature of the problem. The VRP is to decide which route needs to be chosen for a fleet of homogeny vehicle such that to minimize the total cost or travelled distance. There are some conditions must be met which varies from one problem to another problem. There are a lot of literatures have been published to discuss the development of methods to be used, applications and variety of its problem structure (see for example, [1]; [2], and [3]).

In classical VRP the vehicle used to deliver demands for the customers is assumed to be homogeny. However, in most practical logistic problems, customer demands are served using a fleet of heterogeneous vehicles (see, e.g., [7]). This situation could create othe problems, such as, fleet dimensioning or composition and it is necessary to decide the trade-off between owning and keeping a fleet and subcontracting. These decisions could involve additional variables, such as, transportation rates, transportation costs and expected demand. In VRP terms the use of heterogeneous vehicles are called heterogeneous VRP (HVRP). Therefore the objective of the HVRP is to find fleet composition and a corresponding routing plan that minimizes the total cost.

II. RELATED WORKS

HVRPs are originally derived from the seminal paper of [4] and have developed into a large fruitful research area, in creating and developing methods and applications. [5] provided a general overview of papers with a particular focus on lower bounding techniques and heuristics. The authors also compared the performance of existing heuristics described

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until 2008 on benchmark instances. [6] presented a review of exact algorithms and a comparison of their computational performance on the capacitated VRP and HVRPs, while [7] reviewed several industrial aspects of combined fleet composition and routing in maritime and road-based transportation. A comprehensive rigorous review for HVRP can be found in [8].

Due to the mathematical structure of the VRP it is logic that the most approach addressed for tackling the HVRP is heuristics. So far, [9] were the first to tackle the HVRP and developed a number of parallel insertions heuristics. In [10], the authors presented a simulated annealing meta heuristic for the HVRPTW. They outperformed the results of [9]. Then [11] developed a linearly scalable hybrid threshold-accepting and guided local search meta heuristic to handle large scale HVRPTW. In [12], the authors designed a trajectory search heuristic to solve a large-scale HVRPTW in which the vehicles are allowed to have multi-trips. [13] use a variable neighborhood-based heuristics for solving the problem. [14] developed a tabu search for solving HVRPTW, and they conducted the approach on a large set of test cases. In [15], the authors proposed a hybrid algorithm for the problem. Their algorithm is composed by an Iterrated Local Search based heuristic and a Set Partitioning formulation an exact based method. [16] described a modified column generation to solve HVRPTW

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Taking into account scheduled time of delivering customers' demand in the routing problems is another variant of the VRP, known as the periodic VRP (PVRP). In PVRP, within a given time horizon, we have a set of customers needs to be delivered once or several times. There would be a set of delivering schedules associated with each customer. A fleet of vehicles is known available and each vehicle leaves the depot, serves a set of customers, when its work shift or capacity is over, returns to the depot. The objective of the problem is to minimize the total length of the routes travelled by the vehicles on the time horizon. This problem has several important real world applications, such as, distribution for bakery companies ([17]), blood distribution as in [18], or apare-parts materials for a manufacture of automobile, as in [3]. In [8], the authors describe a case for periodic maintenance of elevators at different locations. Further case studies concerning waste collection and road sweeping can be found in [7] and [6]. In [12], the authors describe a milk collection problem as it is important that the goods are collected when fresh. In [19], the authors implement periodic capacitated PVRP for retail distribution of fuel oils.

A comprehensive survey on PVRP can be found in [20]. Most of the works present heuristic approaches, nevertheless [4] proposed an exact method based on a set partitioning integer linear programming formulation of the problem. The first work on the PVRP is proposed by Hoff, et al., [7]. Then there are other works by Braysy, et al., [10], and Rusell and Igo, [21]. In [19], the authors provide a two-phase heuristic that allows escaping from local optima. They use an integer linear program to assign visit day combinations to customers in order to initialize the system. Moreover, the capacity limit of the vehicles is temporarily. [9] addressed a combined of heuristic and exact method for solving PVRP.

The service company focused in this paper has various type of vehicle in its operation to deliver demands for customers. Each type of vehicle has different capacities to deliver demand for customers peridically. The problem faced by the company can then be called periodic heterogeneous VRP. Due to the condition of the customers' demand to be delivered the company put a strict time limit in terms of the arrival time to each customer. This condition has an impact to the delivery cost.

III. PROBLEM DESCRIPTION

This paper considers daily scheduling problem of several vehicles with different capacity of a food material enterprise to deliver container of food materials (FM) to a customer periodically..

In VRP, it is often to visualize the problem graphically. Therefore, the distribution network can be described as a directed graph, in which the set of customers are represented as nodes N and the link of the network is represented with set of edges E. Then we can write $\mathrm{N} = \{1, \ldots, \mathrm{n}\}$ as the set of customers to be served. We can say that $\mathrm{N} = \{0\}$ represent depot (the center for distribution). The set of edges $\mathrm{E} = \{(i, j) : \mathrm{i}, \mathrm{j} \in \mathrm{N}, \mathrm{i} \neq \mathrm{j}\}$ states the set of route to be travelled by the vehicles used. It is necessary that for each route $(i, j) \in \mathrm{E}$ a travel cost c_{ij} is defined. All vehicle

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to be used are located at the distribution center. Let N_c be the set of customers' vertex, it is assumed that each customer has a known fixed daily demand q_i (≥ 0), a service time s_i (≥ 0), and a limited service time windows $[a_i, b_i]$ At the depot $q_i = q_n = 0$ and also for the service time $s_i = s_n = 0$.

Now we need to define for the periodic case. In the PVRP it is not true to assign directly customers to vehicles, as it is normally done in VRP model. We must firstly assign a customer to an allowable delivery combination. Therefore it is necessary to define the periodic time (hours or days or whatsoever) for the customer and then assign him/her to an available delivery vehicle on each of the chosen time period.

To formulate the model, firstly we denote T as the horizon of time period. Let K denote the set of vehicles. For each vehicle $k \in K$, let Q_k denote the capacity of vehiclke k. As this is a periodic problem, it is then necessary to define the number of distinct delivery combinations, which is denoted with *ND*. Along with that, let S_i be the set of allowable delivery combinations for customer *i*.

Let a fleet of K vehicles composed by m different type of vehicles, each with capacity Q_m is located at the depot. The number of vehicles available for vehicle type m is n_m. A vehicle of type m which depends on the load capacity Q_m leaves the depot to serve customer's demand with route $\{(i, j), i \in V, j \in V \setminus \{0\}\}$. On the arrival of customer $j\,(j \in V \setminus \{0\})$, the vehicle delivers the demand D_j of FM to the customer. The vehicle visits a number of customers (at most once) and after finishing its job returns to the depot. It is assumed that the travelling cost is a fixed cost which consists of cost per travel unit (km) and cost per load unit (kg), which actually depends on the distance of location. This travelling cost is denoted with $c_{ij}\,(i \in V, j \in V \setminus \{0\})$. The other cost incurred in the problem is a penalty cost, (pc_i), due to the lateness of the arrival to a customer $i\,(i \in V \setminus \{0\})$ and/or the departure from a customer $i\,(i \in V \setminus \{0\})$.

At the depot, a time window for vehicles to leave and to return to depot is given by $[a_0, b_0]$. The arrival time of a vehicle at customer i is denoted by a_i and its departure time is b_i. Each route originates and terminates at the central depot and must satisfy the time window constraints, i.e., a vehicle cannot start servicing customer i before a_i and after b_i however, the vehicle can arrive before a_i and wait for service.

IV. MATHEMATICAL MODEL FORMULATION

The decision variables consist of binary variables and continuous variables.

Binary variables.

$x_{ojk}^{t} = 1$ if vehicle $k \in K$ travels from depot to customer $j \in V \setminus \{0\}$ at time schedule $t \in T$

otherwise 0

$x_{ijm}^{t} = 1$ if vehicle of type $m \in K_m$ is to deliver from customer $i \in V$ to customer $j \in V \setminus \{0\}$, $i \neq j$ at time schedule $t \in T$, otherwise 0.

z_{ijk}^{t} = 1 if vehicle $k \in K$ does not arrive in time at customer $j \in V \setminus \{0\}$ from $i \in V, i \neq j$, at time schedule $t \in T$ otherwise 0.

Non-negative continuous variables

l_{im} Arrival time for vehicle type $m \in K_m$ at customer $i \in V \setminus \{0\}$

u_{im} Duration of service of vehicle type $m \in K_m$ at customer $i \in V \setminus \{0\}$

The main target of the manager is to use the available vehicle for each type efficiently, such that the total cost is minimized, the customers' order are met, and the periodic scheduled time is satisfied.

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The objective or the problem is to minimize the total cost which consists of traveling cost of all vehicle used, the dispatching cost and the penalty cost in the planning horizon time of a day.

$$Minimize\ \ dc \sum_{j\in V\backslash\{0,n\}} \sum_{k\in K} x_{0jk} + \sum_{(i,j)\in V\backslash\{0\}, i\neq j} \sum_{m\in K_m} c_{ijm} x_{ijm} + \sum_{i\in V\backslash\{0\}} \sum_{j\in V\backslash\{0\}, i\neq j} \sum_{k\in K} pc_i z_{ijk} \quad (1)$$

Subject to

$$\sum_{k\in K} x^t_{0jk} = 1, \qquad \forall j \in V\backslash\{0\}, t\in T \quad (2)$$

$$\sum_{i\in V\backslash\{0\}} x^t_{i0k} = 1 \qquad \forall k\in K, t\in T \quad (3)$$

$$\sum_{i\in V\backslash\{0\}} \sum_{k\in K} x^t_{ijk} = 1, \qquad \forall j\in V\backslash\{0\}, i\neq j, t\in T \quad (4)$$

$$\sum_{i\in V\backslash\{0\}} x^t_{ihk} = \sum_{j\in V\backslash\{0\}} x^t_{hjk} \qquad \forall h\in V\backslash\{0\}, \forall k\in K, i\neq h\neq j, t\in T \quad (5)$$

$$\sum_{j\in V\backslash\{0\}} x^t_{1jk} \leq 1; \qquad \forall k\in K, t\in T \quad (6)$$

$$\sum_{i\in V\backslash\{0\}, i>1} x^t_{i1k} \leq 1; \qquad \forall k\in K, t\in T \quad (7)$$

$$\sum_{i\in V} D_i \sum_{j\in V\backslash\{0\}} x_{ijm} \leq Q_m \qquad \forall m\in K_m \quad (8)$$

$$l_{im} \leq a_i \sum_{j\in V_c} x_{ijm}; \qquad \forall m\in K_m, i\in V \quad (9)$$

$$a_i \sum_{j\in V\backslash\{0\}} x_{ijm} \leq l_{im} + u_{im} \leq b_i \sum_{j\in V\backslash\{0\}} x_{ij} \qquad \forall m\in K_m, \forall i\in V\backslash\{0\}, i\neq j \quad (10)$$

$$\sum_{j\in V\backslash\{0\}} x_{ojm} \leq n_m \qquad \forall m\in K_m \quad (11)$$

$$\sum_{i,j\in V\backslash\{0\}, i\neq j} \sum_{k\in K} z_{ijk} = 1 \quad (12)$$

$$l_{ik} \geq a_i z_{ijk} \qquad \forall i,j\in V\backslash\{0\}, i\neq j, \forall k\in K \quad (13)$$

$$u_{ik} \leq b_i z_{ijk} \qquad \forall i,j\in V\backslash\{0\}, i\neq j, \forall k\in K \quad (14)$$

Constraints (2) and (3)) are to ensure that exactly one vehicle regardless their type enters and departs from every customer and from the central depot.and comes back to the depot. Constraints (4) is necessary to make sure that each customer is served by one and only one vehicle. Constraints (5) is to ensure that the same vehicle enters and leaves each customer and proceeds the continuity of route. Constraints (6) and (7) present the availability of vehicles by bounding the number of route, related to vehicle k for each type, directly leaving from and returning to the central depot, not more than one, respectively. Constraints (8) ensure that each delivery does not exceed the capacity of each type of vehicle. Constraints (9) and (10) present time window as the committed scheduled time-dependent for each customer Constraint (11) guarantees that the number availability of active vehicle does not exceed the number of vehicle available at the central depot. The conditions for penalty are expressed in Constraints (12), (13), and (14).

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V. THE ALGORITHM

Stage 0.
Solve the relaxation of model (1) to (14), as a large scale linear programming.
If all variables in the continuous optimal solution is integer valued, the optimal integer solution to the problem (1) to (14) is found. STOP.
Otherwise go to Stage 1.
Stage 1.
In this Stage we use a hybrid strategy, heuristic approach and exact method.
Step 1. Heuristic

Get row $i*$ the smallest integer infeasibility, such that $\delta_{i*} = \min\{ f_i, 1 - f_i \}$

(This choice is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Exact method
Do a pricing operation

$$v_{i*}^T = e_{i*}^T B^{-1}$$

Step 3. Calculate $\sigma_{ij} = v_{i*}^T \alpha_j$

With j corresponds to

$$\min_j \left\{ \left| \frac{d_j}{\alpha_{ij}} \right| \right\}$$

Calculate the maximum movement of nonbasic j at lower bound and upper bound.

Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column $j*$ is to be increased form LB or decreased from UB. If none go to next $i*$.

Step 4.

Solve $B\alpha_{j*} = \alpha_{j*}$ for α_{j*}

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic $j*$ from its bounds.
Step 6. Exchange basis
Step 7. If row $i* = \{\varnothing\}$ go to Stage 2, otherwise
Repeat from step 1.
Stage 2. Heuristic
Pass1 : adjust integer infeasible superbasics by fractional steps to reach complete integer feasibility.
Pass2 : adjust integer feasible superbasics. The objective of this phase is to conduct a highly localized neighbourhood search to verify local optimality.

VI. CONCLUSION

The food material enterprise has a lot of customers to be served with a variety of volume of food material package . Therefore the company needs several type of vehicle to carry out to fulfill the customers demand. The other thing considered in this paper is the periodic schedule of customers' demand. This paper is then to develop a mathematical model of Periodic Heterogeneous Vehicle Routing with Time Windows Problem This model is used for solving a food material problem of a company located in Medan city, Indonesia. The result model is in the form of mixed integer linear programming problem. We solve the model using a nearest feasible neighborhood search hybrid algorithm.

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REFERENCES

[1] Cordeau J. F, G. Laporte, J.-Y. Potvin, and M.W.P. Savelsbergh, "Transportation on demand", In C. Barnhart and G. Laporte, editors, Transportation, Handbooks in Operations Research and Management Science, pages 429–466, Elsevier, Amsterdam, 2007
[2] Toth, P., Vigo, D. (2nd Eds.), "Vehicle routing : Problems, Methods, and Applications, Society for Industrial and Applied Mathematics & Mathematical Optimization Society", Philadelphia PA, 2014
[3] Drexl M, "Applications of the vehicle routing problem with trailers and transhipment", Technical Report LM-2011-06, Johannes Gutenberg University Mainz, 2011
[4] Golden, B., Assad, A., Levy, L., & Gheysens, F. G., "The fleet size and mix vehicle routing problem". Computers and Operations Research, 11, pp. 49–66, 1984
[5] Baldacci, R., Battarra, M., Vigo, D, "Routing a heterogeneous fleet of vehicles", In B. L. Golden, S. Raghavan, E. A. Wasil, eds. The Vehicle Routing Problem: Latest Advances and New Challenges. (pp. 1–25). Springer, New York, 2008
[6] Baldacci, R., Toth, P., Vigo, D, "Exact algorithms for routing problems under vehicle capacity constraints", Annals of Operations Research 175, 213–245, 2010
[7] Hoff, A., H. Andersson, M. Christiansen, G. Hasle and A. Lokketangen, "Industrial aspects and literature survey: fleet composition and routing," Computers & Operations Research, vol. 37, no. 12, pp. 2041-2061, 2010
[8] Koc, C.,T. Bektas, O. Jabali, G. Laporte., "Thirty years of heterogeneous vehicle routing", European Journal of Operational Research, 249, pp. 1-21, 2016
[9] Liu, F. H., Shen, S. Y., "The fleet size and mix vehicle routing problem with time windows", Journal of the Operational Research Societry, (50)7, pp. 721-732, 1999
[10] Braysy, O., Dullaert, W., Hasle, G., Mester, D., Gendreau, M., "An effective multi-restart deterministic annealing metaheuristic for the fleet size and mix vehicle routing problem with time windows", Transportation Science 42 (3), 371–386, 2008
[11] Braysy, O., Porkka, P.P., Dullaert, W., Repoussis, P.P., Tarantilis, C.D., "A well scalable metaheuristic for the fleet size and mix vehicle routing problem with time windows", Expert System and Applications 36 (4), 8460–8475, 2009
[12] Coelho V N, Grasas A, Ramalhinho H, Coelho I M, Souza M J F, Cruz R C., "An ILS-based algorithm to solve a large-scale real heterogeneous fleet VRP with multi-trips and docking constraints", European Journal of Operational Research, 250, 367-376, 2016
[13] Imran, A., Salhi, S., Wassan, N. A., "A variable neighborhood-based heuristic for the heterogeneous fleet vehicle routing problem", European Journal of Operational Research 197, 509–518, 2009
[14] Jiang J., Ng K.M., Poh K.L., Teo K.M., "Vehicle routing problem with a heterogeneous fleet and time windows", Expert Systems with Applications, 41, 3748–3760, 2015
[15] Subramanian, A., P. H. V. Penna, E. Uchoa, L. S. Ochi., "A hybrid algorithm for the Heterogeneous Fleet Vehicle Routing Problem", European Journal of Operational Research, 221, pp. 285-295, 2012
[16] Yousefikoshbakht M, Dolatnejad A, Didehvar F, Rahmati F., "A modified column generation to solve the heterogeneous fixed fleet open vehicle routing problem", Journal of Engineering, Vol. 2016, pp. 1-12, 2016
[17] Pacheco, J., A. Alvarez, I. Garcia, F. Angel-Bello, "Optimizing vehicle routes in a bakery company allowing flexibility in delivery dates", Journal of the Operations Research Society 63, 569-581, 2012
[18] Cacchiani, V., V. C. Hemmelmayr, F. Tricoire, "A set covering based heuristic algorithm for the periodic vehicle routing problem", Discrete Applied Mathematics, 163, 53-64, 2014
[19] Blakely, F., Bozkaya, B., Cao, B., Hall, W., Knolmajer, J., "Optimizing periodic maintenance operations for Schindler Elevator Corporation", Interfaces 33(1), 67—79, 2003
[20] Alegre, J., M. Laguna, J. Pacheco., "Optimizing the periodic pick-up of raw materials for a manufacture of auto parts". European Journal of Operational Research, 179, 736-746, 2007
[21] Russell, R., Igo, W., "An assignment routing problem", Networks 9(1), 1—17, 1979