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A Combined Method for Capacitated Periodic Vehicle Routing Problem with Strict Time Windows Considering Pickup and Delivery

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Abstract—The paper develops a model for the optimal management of periodic deliveries of a given commodity with known capacity called Capacitated Periodic Vehicle Routing Problem (CPVRP). Due to the large number of customers, it is necessary to incorporate strict time windows, and pick-up and delivery in the periodic planning.. The goal is to schedule the deliveries according to feasible combinations of delivery days and to determine the the routing policies of the vehicles. The objective is to minimize the sum of the costs of all routes over the planning horizon. We model the problem as a large-scale linear mixed integer program and we propose a combined approach to solve the problem.

Keywords-*Capacitated vehicle routing problem, Periodic scheduling, Integer programming, combined method*

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I. Introduction

A well known combinatorial optimization problem which requires the determination of an optimal set of routes used by a fleet of vehicles with known capacity to serve a set of customers is called capacitated vehicle routing problem (CVRP). Generally, using graph concept, CVRP can be defined as follows. Let $G = (V, A)$ be a connected digraph, where $V = \{v_1,v_2,\ldots,v_n\}$ is a set of vertices and $A \subseteq \{(v_i, v_j) : i \neq j, v_i, v_j \in V\}$ is the arc set. The problem is to find an optimal set of routes, composed of a cyclic linkage of arcs starting and ending at the depot, in order to satisfy demands a given set of customers at vertices. The objective is to minimize the total travel cost, which is proportional to the travel times or distances and other operational costs. Due to its important role in the logistic system, the research domain of CVRP has been widely explored. [1] propose a model of CVRP for deploying Product-Service-System configuration in urban goods transport. A number of rigorous and up-to-date survey which discusses all the different variations of the vehicle-routing problem and solution methodologies (exact and heuristics) can be found in [2] and [3].

Whenever the delivery for each customer is done periodically, within a given time horizon, the CVRP is then called capacitated periodic VRP (CPVRP). Therefore it is necessary to have a visiting schedules associated with each customer. A fleet of vehicles is available and each vehicle leaves the depot, serves a set of customers, when its work shift or capacity is over, returns to the depot. The problem is to minimize the total length of the routes travelled by the vehicles on the periodic time horizon. Several real–world applications deal with periodic delivery operations over a given time horizon can be found in distribution for bakery companies [4] , blood product distribution [5] or pick-up of raw materials for a manufacture of automobile parts [6]. More comprehensive applications of CPVRP can be found in [7]

Due to the complexity of the problem most of the works present heuristic approaches, nevertheless [8] proposed an exact method. [9] addressed a combined of heuristic and exact method for solving PVRP. Early formulations of the PVRP were developed by [10]. But the first mathematical formulation were given by [11] and [12]. [13] use the idea of the generalized assignment method proposed by [14] and assign a visiting schedule to each vertex. Eventually a heuristic for the VRP is applied to each delivery day. [15] developed a heuristic organized in four phases. Solution methods in these papers have focused on two-stage (construction and improvement) heuristics. [16] present another algorithm: The solution algorithm is a travelling salesman heuristic which, differently from the other heuristics, may allow infeasible solutions during the search process. Similarly, good results were obtained in the more recent work of [17], [18], and [19] who provide specific practical applications of the PVRP. [20] propose a flexible periodic vehicle routing problem.

[21] address a vehicle routing problem where the service to the customers has to be provided over multiple periods. It is assumed that customers must be served with a certain frequency and according to a given schedule and they should receive a fixed quantity at each visit. The problem is to choose the visit schedule for each customer and to organize vehicle routes. Another broad class of routing problems dealing with multi-period plans is the class of Inventory Routing Problems (IRPs) [22], [23], [24].

[25] present PVRP with time windows (PVRPTW). The problem requires the generation of a limited number of routes for each day of a given planning horizon. The objective is to minimize the total travel cost while satisfying several constraints

This paper concerns with a comprehensive model for the CPVRP incorporated with strict time windows, considering pick-up and delivery (CPVRPPD). We propose a mixed integer programming formulation to model the problem. A feasible neighborhood heuristic search is addressed to get the integer feasible solution after solving the continuous model of the problem.

Section 2 describes the mathematical model of capacitated VRP. Section 3 presents the model of periodic VRP. The mathematical model for capacitated periodic VRP with pick up and delivery (CPVRPPD) is described in Section 4. The algorithm is described in Section 5. Finally Section 6 describes the conclusions.

II. MATHEMATICAL MODEL FOR CVRP

As can be found in many literatures, CVRP can be define using graph concept. Let a complete undirected graph $G = (V, E)$ is well defined, where $V = \{v_0, v_1, ..., v_n\}$ is the set of nodes and E is the set of arcs. Node $v_0 \in V$ represents a depot or a distribution centre where, in VRP concept, a fleet of K vehicles is located. It is assumed that there are k vehicles with identical capacity, namely Q, are available (ready to be used anytime). The nodes $v_i \in V \setminus \{v_0\}, i \in (1, ..., n)$ are assigned for customers to be served with demand q_i, $(q_i > 0)$. While arcs $(i, j) \in E$ represent route to be travelled by the vehicles from a node $v_i \in V$ to other node $v_j \in V$, with $i \neq j$, where travelling costs c_{ij} are incurred.

The optimal decision to be made for CVRP is to find the route for the vehicle in which all customers are visited with their corresponding demands considering the limited cumulative capacity Q, such that the total operation costs are minimized.

Before we formulate the mathematical model of CVRP, it is necessary to define two binary variables.

x_{ijm} = 1, if vehicle $m \in K$ visits customer v_j immediately after serving customer v_i with $(i \neq j)$

= 0, otherwise

y_{im} = 1, if a vehicle $m \in K$ is assigned to customer v_i

= 0, otherwise

The objective is to minimize the travel costs

$$Minimize \sum_{i \in V} \sum_{j \in V \setminus \{0\}} \sum_{m \in K} c_{ij} x_{ijm} \tag{1}$$

Subject to a set constraints

$$\sum_{m \in K} y_{im} = 1, \qquad \forall i \in V \setminus \{0\} \tag{2}$$

$$\sum_{m \in K} y_{0m} \leq k \tag{3}$$

$$\sum_{m \in K} x_{0jm} = 1, \qquad \forall j \in V \setminus \{0\} \tag{4}$$

$$\sum_{m \in K} \sum_{j \in V \setminus \{0\}} x_{ijm} = 1, \qquad \forall i \in V \tag{5}$$

$$\sum_{i \in V} x_{ijm} - \sum_{i \in V} x_{jim} = 1; \qquad \forall j \in V \setminus \{0\}, \forall m \in K \tag{6}$$

$$\sum_{i \in V} q_i y_{im} \leq Q, \qquad \forall m \in K \tag{7}$$

$$\sum_{j \in V} x_{ijm} = \sum_{j \in V} x_{jim} = y_{im}, \qquad \forall i \in V, \forall m \in K \tag{8}$$

$$\sum_{v_i \in S} \sum_{v_j \in S} x_{ijm} \leq |S| - 1, \qquad \forall m \in K; S \in V \setminus \{0\}; S \geq 2 \tag{9}$$

$$y_{im} \in \{0,1\}, \qquad \forall i \in V; \forall m \in K \tag{10}$$

$$x_{ijm} \in \{0,1\}, \qquad \forall i \in V; \forall j \in V; \forall m \in K \tag{11}$$

Constraint (2) ensures that only one vehicle is assigned to a customer. Constraint (3) is to make sure that the number of vehicle used should be at least the available vehicles. Constraints (4) and (5) state that exactly one vehicle enters and departs from each customer and from depot and then come back to the depot. A flow conservation equation is presented in Constraint (6). To limit the capacity to be delivered is modeled in Constraint (7). The structures of routes and the elimination of sub-tours are, respectively, given in Constraints (8) and (9). Constraints (10) and (11) present the nature of the decision variables.

III. MODELS OF THE PVRP

In the PVRP, it is not allowed to assign directly customers to directly to vehicles instead customers are visited a preset number of times over the period with a schedule that is chosen from a menu of schedule options.

To formulate the model, firstly we denote T as the horizon of days time period. Let K denote the set of vehicles. For each vehicle $m \in K$, let Q_k denote the capacity of vehicle k. In a periodic problem, it is necessary to define the number of distinct delivery combinations, ND. Relating with that, let S_i be the set of allowable delivery combinations or menu of schedule for customer i. The parameter a_{st} links schedules to days, where a $a_{st} = 1$ if day $t \in T$ is in schedule $s \in S$ and $a_{st} = 0$ otherwise. Each schedule $s \in S$ has an associated visit frequency γ^s measured by the number of days in the schedule: $\gamma^s = \Sigma_{t \in T} a_{st}$. For a given schedule option s, the headway between visits is defined in terms of the visit frequency as $H^s = 1/\gamma^s$.

Denote the set of n customers (/nodes) by $N = \{1, 2, \cdot, n\}$. Denote the depot by $\{0, n+1\}$ Each vehicle starts from $\{0\}$ and terminates at $\{n+1\}$. Each customer $i \in N$ specifies a set of days to be visited, denoted by $T_i \subseteq T$. On each day $t \in T_i$, customer $i \in N$ requests service with demand of q_i^t in weight and p_i^t in volume, service duration d_i^t and time window $[a_i, b_i]$. Note that, for the depot $i \in \{0, n+1\}$ on day t, we set $q_i^t = p_i^t = d_i^t = 0$. The travel time between customer i and j is given by c_{ij}.

IV. MATHEMATICAL FORMULATION OF CPVRPPD

Let K denote the set of vehicles. For each vehicle $k \in K$, let Q_k and P_k denote the capacity in weight and in volume, respectively. We assume the number of vehicles equals to the number of drivers. Denote the set of n customers (/nodes) by $N = \{1, 2, \cdot, n\}$. Denote the depot by $\{0, n+1\}$. Each vehicle starts from $\{0\}$ and terminates at $\{n+1\}$. Each customer $i \in N$ specifies a set of days to be visited, denoted by $T_i \subseteq T$. On each day $t \in T_i$, customer $i \in N$ requests service with demand of q_i^t in weight and p_i^t in volume, service duration d_i^t and time window $[a_i, b_i]$. Note that, for the depot $i \in \{0, n+1\}$ on day t, we set $q_i^t = p_i^t = d_i^t = 0$.

Notations used are given as follows :

Set:

T	The set of workdays in the planning horizon,
K	The set of vehicles,
N	The set of customers,
N_0	The set of customers and depot $N_0 = \{0, n + 1\} \cup N$,
K_i	The set of preferable vehicles for customer $i \in N$,
T_i	The set of days on which customer $i \in N$ orders,
ND	The number of distinct delivery combinations
S	The set of allowable delivery combinations, in this case $S \subseteq [1,\ldots, \mathrm{ND}]$

Parameter:

Q_k	The weight capacity of vehicle $k \in K$,
P_k	The volume capacity of vehicle $k \in K$,
c_{ij}	The travel time from node $i \in N_0$ to node $j \in N_0$,
$[a_i, b_i]$	The earliest and the latest visit time at node $i \in N_0$,
d_i^t	The service time of node $i \in N_0$ on day $t \in T_i$,
q_i^t	The weight demand of node $i \in N_0$ on day $t \in T_i$,
p_i^t	The volume demand of node $i \in N_0$ on day $t \in T_i$,
e	The extra service time per pallet when a non-preferable vehicle is used,
$[g_l^t, h_l^t]$	The start time and the latest ending time of driver $l \in D$ on day $t \in T$,
α_i^t	Pick up quantity for customer i on day $t \in T_i$,
β_t^i	Delivery quantity for customer I on day $t \in T^i$,

Variables:

x_{ijm}^t	Binary variable indicating whether vehicle $m \in K$ travels from node $i \in N_0$ to $j \in N_0$ on day $t \in T$,
w_i^s	Binary variable indicating whether customer $i \in N_0$ is served with s delivery combination,
y_{im}^t	Binary variable stating whether customer i is visited on day t using vehicle m
v_{im}^t	The time at which vehicle $m \in K$ starts service at node $i \in N_0$ on day $t \in T$,
θ_{jm}^t	Number of pick up demand of customer j served by vehicle $m \in K$ on day $t \in T$
σ_{jm}^t	Number of delivery demands of customer j served by vehicle $m \in K$ on day $t \in T$

The CPVRPPD problem is to design route to deliver customers demand periodically such that to minimize the travelling cost of all vehicle used and to satisfy several constraints, such as, a strict time windows between each delivery, capacity of vehicles and logical conditions.

Firstly, we formulate the objective function to minimize the travelling cost

$$Minimize \sum_{t\in T}\sum_{i\in N_0}\sum_{j\in N_0}\sum_{k\in K} c_{ij} x_{ijk}^t \tag{12}$$

Subject to a set constraints

$$\sum_{m\in K} y_{im} = 1, \qquad \forall i \in V \setminus \{0\} \tag{13}$$

$$\sum_{m\in K} y_{0m} \le k \tag{14}$$

$$\sum_{m\in K} x_{0jm} = 1, \forall j \in V \setminus \{0\} \tag{15}$$

$$\sum_{m\in K}\sum_{j\in\{0\}} x_{ijm} = 1, \forall j \in V \tag{16}$$

$$\sum_{i\in V} x_{ijm} - \sum_{i\in V} x_{jim} = 1; \qquad \forall j \in V \setminus \{0\}, \forall m \in \mathrm{K} \tag{17}$$

$$\sum_{i\in V} q_i y_{im} \le Q, \qquad \forall m \in K \tag{18}$$

$$\sum_{j\in V} x_{ijm} = \sum_{j\in V} x_{jim} = y_{im}, \qquad \forall i \in V, \forall m \in K \tag{19}$$

$$\sum_{v_i\in S}\sum_{v_j\in S} x_{ijm} \le |\mathrm{S}| - 1, \quad \forall m \in K; S \in V \setminus \{0\}; \mathrm{S} \ge 2 \tag{20}$$

$$\sum_{s\in S} w_i^s = 1, \qquad \forall i \in N_0 \tag{21}$$

$$y_i^t = \sum_{s\in S} w_i^s z_{st}, \qquad \forall t \in T, \forall i \in N^0 \tag{22}$$

$$\sum_{k\in K} x_{0j}^{k} = 1, \qquad \forall j \in N \tag{23}$$

$$\sum_{k\in K} x_{ijk}^{t} \le \frac{y_i^t + y_j^t}{2}, \qquad \forall t \in T, \forall i, j \in N, (i \neq j) \tag{24}$$

$$\sum_{j\in N} x_{ijm}^{t} = \sum_{j\in N_0} x_{ijk}^{t}, \qquad \forall m \in K, \forall i \in N_0, \forall t \in T \tag{25}$$

$$\sum_{m\in K}\sum_{i\in N_0} x_{ijm}^{t} = y_j^t, \qquad \forall j \in N, \forall t \in T \tag{26}$$

$$\sum_{j\in N} x_{0jm}^{t} \le 1, \qquad \forall m \in K, \forall t \in T \tag{27}$$

$$\sum_{m\in K}\sum_{j\in N_0} x_{ijm}^{t} = 1 \ \forall\, i \in N, t \in T_i \tag{28}$$

$$\sum_{i\in N}\sum_{j\in N_0} q_i^t x_{ijm}^t \le Q_m \ \forall\, m \in K, t \in T \tag{29}$$

$$\sum_{i\in N}\sum_{j\in N_0} p_i^t x_{ijm}^t \le P^m \ \forall\, m \in K, t \in T \tag{30}$$

$$b_i \ge v_{im}^t \ge a_i \forall i \in N,\ m \in K, t \in T_i \tag{31}$$

$$v_{0m}^t \ge \sum_{l\in D}(g_l^t \cdot y_{lm}^t) \forall m \in K, t \in T \tag{32}$$

$$v_{n+1,m}^t \le \sum_{l\in D}(h_l^t \cdot y_{im}^t) \forall m \in K, t \in T \tag{33}$$

$$\sum_{m\in K} \theta_{jm}^t = \alpha_j^t \ \ \forall j \in N, t \in T \tag{34}$$

$$\sum_{m\in K} \sigma_{jm}^t = \beta_j^t \ \ \forall j \in N, t \in T \tag{35}$$

$$x_{ijm}^t, w_i^s, y_{im}^t \in \{0,1\} \ \forall i, j \in N_0,\ m\in K, t \in T, \ \ s \in S \tag{36}$$

$$v_{im}^t \ge 0, \forall i \in N_0, \ \ m \in K, t \in T \tag{37}$$

$$\theta_{jm}^t, \sigma_{jm}^t \in \{0,1,2,\ldots\} \ \forall j \in N, m\in K, t \in T \tag{38}$$

The explanation of Constraints (13) to (20) can be found in the explanation of Constraints (2) to (9).

Constraint (21) is to make sure that one and only one combination is chosen for each customer. In Constraint (22), we schedule service days for each customer which related to the chosen service combination. Constraint (23) is to guarantee that only one vehicle will leave the depot (distribution center) to serve a customer. In Constraint (24), we define that the route chosen by a vehicle is related to visited-day of a customer. In order to ensure that vehicles will visit and leave a node of customer on the same day it is necessary to have Constraint (25). Constraint (26) states that to serve a customer should be on the selected day. Constraint (27) describes that only one vehicle must be used once in the time period. Constraints (28) state that each customer must be visited by one vehicle on each of its delivery days. Constraints (29-30) guarantee that the vehicle capacities are respected in both weight and volume. Constraints (31) is to make sure the services start within the customers' time window

Constraints (32-33) are to ensure that the starting time and ending time of each route must lie between the start working time and latest ending time of the assigned vehicle. Constraints (34–35) define the pick up and delivery for each customer. Constraints (36-38) define the binary and the continuous variables used in this model.

V. THE ALGORITHM

Stage 1.

Step 1. Get row i^* the smallest integer infeasibility. (This step is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Do a pricing operation

$$v_{i*}^T = e_{i*}^T B^{-1}$$

Step 3. Calculate $\sigma_{ij} = v_{i*}^T \alpha_j$ as the maximum movement of nonbasic j, if any

With j corresponds to

$$\min_j \left\{ \left| \frac{d_j}{\alpha_{ij}} \right| \right\}$$

Otherwise go to next non-integer nonbasic or superbasic j (if available). If none go to next i^*.

Step 4.

Solve $B\alpha_{j*} = \alpha_{j*}$ for α_{j*}

Step 5. Do ratio test in order to stay feasible due to the releasing of nonbasic j^* from its bounds.

Step 6. Exchange basis

Step 7. If row $i^* = \{\varnothing\}$ go to Stage 2, otherwise Repeat from step 1.

Stage 2. Pass1 : adjust integer infeasible superbasics by fractional steps to reach complete integer feasibility.

Pass2 : adjust integer feasible superbasics. The objective of this phase is to conduct a highly lovalized neighbourhood search to verify local optimality.

VI. CONCLUSIONS

This paper was intended to develop efficient technique for solving one of the most economic importance problems in optimizing transportation and distribution systems. The aim of this paper was to develop a model of Capacitated Periodic vehicle Routing with strict Time Windows, Pick-up and Delivery Problem This problem has additional constraint which is the limitation in the delivery periodic time schedule. The proposed algorithm employs nearest neighbor heuristic algorithm for solving the model.

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